

Laws and Theorems of Boolean Algebra

Operations with 0 and 1:

$$1. \quad X + 0 = X$$

$$2. \quad X + 1 = 1$$

$$1D. \quad X \cdot 1 = X$$

$$2D. \quad X \cdot 0 = 0$$

Idempotent laws

$$3. \quad X + X = X$$

$$3D. \quad X \cdot X = X$$

Involution law:

$$4. \quad (X')' = X$$

Laws of complementarity:

$$5. \quad X + X' = 1$$

$$5D. \quad X \cdot X' = 0$$

Commutative laws:

$$6. \quad X + Y = Y + X$$

$$6D. \quad X \cdot Y = Y \cdot X$$

Associative laws:

$$7. \quad (X + Y) + Z = X + (Y + Z) \\ = X + Y + Z$$

$$7D. \quad (XY)Z = X(YZ) = XYZ$$

Distributive laws:

$$8. \quad X(Y + Z) = XY + XZ$$

$$8D. \quad X + YZ = (X + Y)(X + Z)$$

Simplification theorems:

$$9. \quad XY + XY' = X$$

$$9D. \quad (X + Y)(X + Y') = X$$

$$10. \quad X + XY = X$$

$$10D. \quad X(X + Y) = X$$

$$11. \quad (X + Y')Y = XY$$

$$11D. \quad XY' + Y = X + Y$$

DeMorgan's laws:

$$12. \quad (X + Y + Z + \dots)' = X'Y'Z' \dots$$

$$12D. \quad (XYZ \dots)' = X' + Y' + Z' + \dots$$

$$13. \quad [f(X_1, X_2, \dots, X_N, 0, 1, +, \cdot)]' = f(X_1', X_2', \dots, X_N', 1, 0, \cdot, +)$$

Duality:

$$14. \quad (X + Y + Z + \dots)^D = XYZ \dots$$

$$14D. \quad (XYZ \dots)^D = X + Y + Z + \dots$$

$$15. \quad [f(X_1, X_2, \dots, X_N, 0, 1, +, \cdot)]^D = f(X_1, X_2, \dots, X_N, 1, 0, \cdot, +)$$

Theorem for multiplying out and factoring:

$$16. \quad (X + Y)(X' + Z) = XZ + X'Y$$

$$16D. \quad XY + X'Z = (X + Z)(X' + Y)$$

Consensus theorem:

$$17. \quad XY + YZ + X'Z = XY + X'Z$$

$$17D. \quad (X + Y)(Y + Z)(X' + Z) \\ = (X + Y)(X' + Z)$$